Computer simulation of stochastically growing interfaces with a conservation law

1990 J. Phys. A: Math. Gen. 23 L919

(http://iopscience.iop.org/0305-4470/23/17/013)

J. Phys. A: Math. Gen. **23** (1990) L919-L922. Printed in the UK

LETTER TO THE EDITOR

Computer simulation of stochastically growing interfaces with a conservation law

Amitabha Chakrabarti

Department of Physics, Kansas State University, Manhattan, KS 66506, USA

Received 6 June 1990

Abstract. We present the first direct numerical study of a model for stochastically growing interfaces with a conserved height-field, proposed by Sun, Guo and Grant. The scaling exponent calculated in the simulation is in excellent agreement with the dynamic renormalization group calculations by the above authors.

The dynamics of stochastically growing interfaces has received considerable attention in recent years [1-6]. One class of problems, which has been studied extensively, includes the Eden [7] and the ballistic-deposition models [8]. These growth models produce compact clusters with a rough interface, the width or thickness of which shows interesting scaling behaviour. Additional interest in the scaling behaviour of the interface width developed when Kardar, Parisi and Zhang [9] proposed a nonlinear stochastic differential equation (hereafter referred to as the KPZ equation) to govern the growth of profiles for the above class of processes [10-13]. The KPZ equation received added attention since it was realized that this equation is closely related to other physical problems such as randomly stirred fluids [14] and directed polymers in random media [15, 16].

Recently, Sun, Guo and Grant (SGG) [17] have considered a situation of a growing interface where the total volume under the interface is conserved. In order to study this case, they have suitably generalized the KPZ equation for a conserved height-field and investigated the dynamics of this new model by using renormalization group methods. The conservation law seems to lead to a different universality class from that discussed by KPZ since scaling exponents for the width of the interface are found to be different from the KPZ model. The above authors have also studied a conserved restricted solid-on-solid model by computer simulation methods and found good agreement between the theoretical scaling exponents and those computed from the simulation of the microscopic model. However, no direct numerical study of the conserved model of SGG has yet been carried out.

In this letter, we present results of the first direct numerical study of the SGG model in two dimensions. We calculate the scaling exponent of the width of the growing interface and find good agreement with the theoretical predictions of SGG.

The KPZ equation for the interface profile is written in terms of a coarse-grained interface height variable $h(\boldsymbol{r}, t)$ in a d-dimensional system as

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\boldsymbol{\nabla} h)^2 + \eta \qquad (1)$$

where ν and λ are constants, and the noise η is a Gaussian distributed stochastic variable of mean $\langle\eta(\boldsymbol{r}, t)\rangle = 0$ and correlations given by

$$\langle\eta(\boldsymbol{r}, t)\eta(\boldsymbol{r}', t')\rangle = 2D\delta^{d-1}(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t') \tag{2}$$

where D is a constant and the vector $\boldsymbol{r}$ defines the $(d-1)$ dimensional space of the 'substrate'. The long-time, large-length scale behaviour of the model can be probed by measuring the width of the interface defined as

$$W(L, t) \equiv \sqrt{\langle h(\boldsymbol{r}, t)^2\rangle - \langle h(\boldsymbol{r}, t)\rangle^2} \tag{3}$$

where L is the linear size of the substrate. The asymptotic behaviour of the interface width $W(L, t)$ is found to obey the scaling relation

$$W = L^{\chi} f(t/L^{z}) \tag{4}$$

where the exponents χ and z are related by

$$\chi + z = 2. \tag{5}$$

The equation (4) leads to two interesting limits: (1) if $L \to \infty$, then for finite but sufficiently large t, $W(t) \sim t^{\chi/z} \equiv t^{\beta}$ and (2) for finite L and $t \to \infty$, $W(L) \sim L^{\chi}$. In two spatial dimensions, the values of the exponents χ and z found by KPZ are given by $\chi = \frac{1}{2}$ and $z = \frac{3}{2}$ (i.e. $\beta = \frac{1}{3}$).

The model proposed by SGG also starts with a coarse-grained height variable $h'(\boldsymbol{r}, t)$ and is described by the following equation for $h'(\boldsymbol{r}, t)$:

$$\frac{\partial h'}{\partial t} = -\nabla^2\left[\nu'\nabla^2 h' + \frac{\lambda'}{2}(\nabla h')^2\right] + \xi(\boldsymbol{r}, t) \tag{6}$$

where $\xi(\boldsymbol{r}, t)$ is a Gaussian noise term given by

$$\langle\xi(\boldsymbol{r}, t)\xi(\boldsymbol{r}', t')\rangle = -2D'\nabla^2\delta^{d-1}(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t'). \tag{7}$$

The conservation law for total h' is obeyed since the right-hand side of (6), including the noise term ξ, can be written as the divergence of some current. The width of the interface $W(L, t)$ satisfies a scaling relation similar to (4):

$$W(L, t) = t^{\chi'} f(t/L^{z'}) \tag{8}$$

but now the scaling exponents χ' and z' are related by

$$\chi' + z' = 4 \tag{9}$$

instead of (5). SGG also found that for $d = 2$, $\chi' = \frac{1}{3}$, $z' = \frac{11}{3}$ and $\beta' = \chi'/z' = \frac{1}{11}$.

Our numerical study of the SGG model ((6) and (7)) is carried out for $d = 2$ (i.e. for a one-dimensional substrate). We choose $\nu' = 1$, $D' = 1$ and consider both $\lambda' = 2$ and $\lambda' = 4$ in our simulations. We consider an $L = 8192$ lattice and carry out the simulations up to $t = 500$ with a time step of $\Delta t = 0.01$. We always start with $h'(\boldsymbol{r}, t) = 0$ everywhere as the initial configuration and average over 50 runs in order to take an ensemble average over the noise.

In figure 1 we show a log-log plot for the width $W(t)$ against t for $\lambda' = 2$. The slope of the straight line in this plot yields β', i.e. χ'/z'. We find that, for $\lambda' = 2$, $\beta' = 0.095 \pm 0.005$. This value of β' is in excellent agreement with the theoretical value of $\frac{1}{11}$, considering our error bars. We have also considered $\lambda = 4$ in our study although in this case the simulation is carried out only until $t = 100$ and was averaged over 20 runs due to unavailability of computer time. We find a similar value of β' is this case ($\beta' = 0.09 \pm 0.01$).

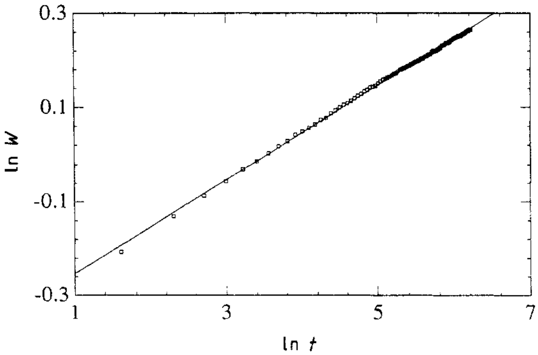

Figure 1. Width $W(t)$ against t in a log-log plot for the SGG model. The full line is the best fit to the data with an exponent $\beta' = 0.095 \pm 0.005$. The theoretical value of β' [17] is $\frac{1}{11}$.

We could not probe the other scaling exponent χ for this model for the following reason. The value of z' in this case is quite large ($z' = \frac{11}{3}$) compared to the KPZ case ($z = \frac{3}{2}$). In order to compute the exponent χ one needs to probe the asymptotic limit $t \gg L^{z'}$ such that $W(L, t) \sim L^{\chi}$ in that limit. Even for a small value of L ($L \sim 100$) this limit is difficult to reach in the computer simulation. However, since the exponent β' agrees quite well with the theoretical calculations, we expect that χ' and z' independently will also be given by their theoretical values.

To conclude, then, we have carried out the first numerical integration study of a model for stochastically growing interfaces in the presence of a conservation law. We find very good agreement of the computed values of the scaling exponent β with those obtained from dynamic renormalization calculations.

Many valuable discussions with Martin Grant and Hong Guo are acknowledged. The computations were carried out using the Cornell National Supercomputer Facility, a resource of the Cornell Theory Center, which is funded in part by the National Science Foundation, New York State, the IBM corporation and the members of the Center's Corporate Research Institute.

References

[1] Family F and Vicsek T 1985 *J. Phys. A: Math. Gen.* **18** L75
[2] Plischke M and Racz Z 1985 *Phys. Rev.* A **32** 3825
[3] Wolf D E and Kertesz J 1987 *Europhys. Lett.* **4** 651
[4] Kim J M and Kosterlitz J M 1989 *Phys. Rev. Lett.* **62** 2289
[5] Amar J G and Family F 1990 *Phys. Rev. Lett.* **64** 543
[6] Yan H, Kessler D and Sander L M 1990 *Phys. Rev. Lett.* **64** 926
[7] Eden M 1961 *Proc. 4th Berkeley Symp. on Mathematical Statistics and Probability* vol 4, ed F Neyman (Berkeley, CA: University of California Press)
[8] Vold M J 1963 *J. Colloid Interface Sci.* **18** 684
Leamy H J, Gilmer G H and Dirks A G 1980 *Current Topics in Materials Science* vol 6, ed E Kalids (Amsterdam: North-Holland)
[9] Kadar M, Parisi G and Zhang Y C 1986 *Phys. Rev. Lett.* **56** 889
[10] Chakrabarti A and Toral R 1989 *Phys. Rev.* B **40** 11419
[11] Guo H, Grossman B and Grant M 1990 *Phys. Rev. Lett.* **64** 1262

[12] Amar J G and Family F 1990 *Phys. Rev.* A **41** 3399
[13] Guo H, Grossman B and Grant M 1990 *Phys. Rev.* A in press
[14] Forster D, Nelson D R and Stephen M J 1977 *Phys. Rev.* A **16** 732
[15] Kardar M and Zhang Y C 1987 *Phys. Rev. Lett.* **58** 2087
[16] Medina E, Hwa T, Kardar M and Zhang Y C 1989 *Phys. Rev.* A **39** 3053
[17] Sun T, Guo H and Grant M 1989 *Phys. Rev.* A **40** 6763